Homework due by 12/6/2025

Consider a sample $\underline{X} = (X_1, X_2, \ldots, X_n)$ from a Poisson distribution $\mathcal{P}(\lambda)$, with density $f(x) = \dfrac{\lambda^x}{x!} e^{-\lambda}$ with $x \in \mathbb{N}$ and $\lambda > 0$

1. Write the likelihood based on the sample $\underline{X}$
2. Compute the Maximum Likelihood Estimator (MLE) $\hat{\lambda}_{MLE}$ of λ given $\underline{X}$
3. Choose (and justify your choice) the functional form of the prior distribution on λ considering one of the distributions presented in class, i.e. Gaussian (Normal), Gamma, Exponential, Beta, Uniform
4. Compute the posterior distribution of λ and a Bayesian estimator $\hat{\lambda}_{PO}$
5. (Optional) Comment about what happens to $\hat{\lambda}_{PO}$ when (like done in class)
   - letting the prior variance going to 0
   - letting the prior variance going to ∞
   - letting the sample size going to ∞
6. Compute the posterior predictive distribution for a next observation X_{n+1} given the sample $\underline{X}$
7. Describe, in your words, why MLE and posterior mean are sound choices as estimators of a parameter
8. Consider the following data which will be modelled by independent and identically distributed Poisson distributions $\mathcal{P}(\lambda)$. The data are (rounded) number of deaths for car accidents for 100,000 inhabitants (Source: Wikipedia):
   - Italy: 5
   - Kuwait: 15
   - United States: 13
   - The Netherlands: 4
   - Brazil: 16

- China: 17
- Egypt: 10
- Japan: 2
- New Zealand: 8
- Romania: 10

9. (Optional) You will do the exercise considering i.i.d. Poisson distributions but you should comment about such choice, i.e. on the assumptions that we are making considering Poisson and considering i.i.d.. Are they completely justified or not? If not, what are the critical aspects?

10. Compute the MLE of λ

11. Based on your experience in your home country, assign the parameters to the prior distribution on λ. As in class, it might be useful to think what is the "physical meaning" of λ and then think of its possible value, to be considered as a mean, and a trust on such assessment, which could be considered as a variance

12. Compute the posterior mean of λ using your elicited prior (elicitation = process leading to the choice of a prior)

13. (Optional) Try some different values of the parameters and comment what happens to the posterior mean

14. (Optional) For those who know R, look for (and use!) the very simple commands which allow you to build a 95% credible interval for λ, using the previous data and your prior distribution